Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2016

MATHEMATICS P1

MARKS: 150

TIME: 3 hours

MATHE1

This question paper consists of 12 pages including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TWELVE questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. Answers only will not necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

7. Number the answers correctly according to the numbering system used in this question paper.

8. Write neatly and legibly.

9. An information sheet with formulae is included at the end of the question paper.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 4x = 12$ (3)

1.1.2 $3x^2 + 2x - 6 = 0$ (correct to two decimal places) (3)

1.1.3 $3^{x^2-1} = \frac{27^{-x}}{3}$ (4)

1.2 Given the equation: $\frac{x - \frac{1}{x}}{1 + \frac{1}{x}} = 1$

1.2.1 For which value(s) of x is the equation undefined? (2)

1.2.2 Solve for x. (5)

[17]

QUESTION 2

2.1 Solve for x and y simultaneously in the following equations:

$$x - y = 3 \quad \text{and} \quad xy = 28$$ (6)

2.2 Solve for x.

$$x^2 \leq 4 + 3x \quad \text{and} \quad x > 0$$ (4)

[10]

QUESTION 3

Given the pattern below:

Row 1				1			
Row 2			3		5		
Row 3		7		9		11	
Row 4	13		15		17		19
⋮							
Row n							

3.1 Determine the value of the first term in Row 80. (5)

3.2 Calculate the sum of the terms in Row 80 only. (4)

[9]

QUESTION 4

4.1 The sum of the first p terms of a sequence of numbers is given by:

$$S_p = p(p+1)(p+2)$$

Calculate the value of T_{10} (3)

4.2 Calculate: $\sum_{r=1}^{p}(2r-1)p$ (4)

[7]

QUESTION 5

The following sequence represents a geometric progression:

$$x\ ;\ x+2\ ;\ \ldots\ \ldots\ldots\ldots\ldots$$

5.1 Write down the third term in terms of x. (2)

5.2 Calculate the value of x if it is given that $S_\infty = -8$. (4)

[6]

QUESTION 6

6.1 The cost price of a car is R635 000. The value of the car depreciates according to the reducing balance method at a rate of 15% p.a. Calculate the value of the car exactly 5 years after it was bought. (3)

6.2 A loan of R50 000 is to be repaid in 48 equal monthly payments. Payments start one month after the loan has been granted. The interest rate on the loan is 16,75% p.a. compounded monthly.

6.2.1 Calculate the value of the equal monthly payments. (4)

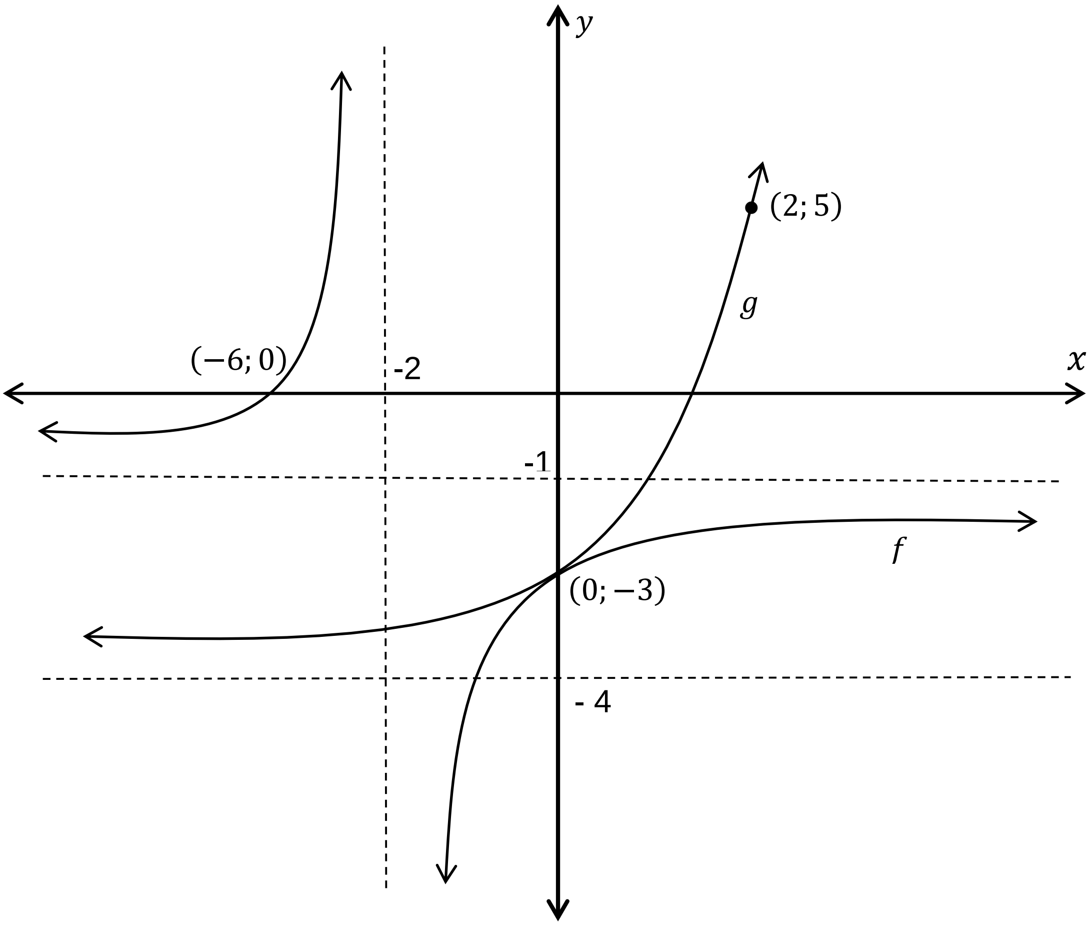
6.2.2 Calculate the outstanding balance immediately after the 30th payment has been made. Give your answer to the nearest Rand. (5)

6.3 How many years will it take for an investment to double its value if the compound interest rate is 14,75% p.a? Leave your answer correct to one decimal digit. (4)

[16]

QUESTION 7

7.1 The sketch below shows the graph of $f(x) = \dfrac{a}{x+p} + q$ and $g(x) = b^x + c$.
The x-intercept of f is at $(-6; 0)$ and the y-intercept of f and g is at $(0; -3)$. The point $(2; 5)$ lies on the graph of g.



7.1.1 For which value(s) of x is $f(x) = g(x)$? (1)

7.1.2 For which values of x is $f(x) < g(x)$? (2)

7.1.3 Write down the equation of the asymptote of g. (1)

7.1.4 Determine the equation of g. (4)

7.1.5 Write down the equations of the asymptotes of f. (2)

7.1.6 Determine the equation of f. (3)

7.1.7 Determine the equations of the axes of symmetry of f. (3)

7.2 Given $f(x) = \log_{\frac{1}{2}} x$.

7.2.1 Write down the equation of f^{-1}, the inverse of f , in the form $y = \dots$ (2)

7.2.2 Draw a neat sketch graph of f^{-1}. Clearly show all intercepts with the axes and any other point on the graph. (3)

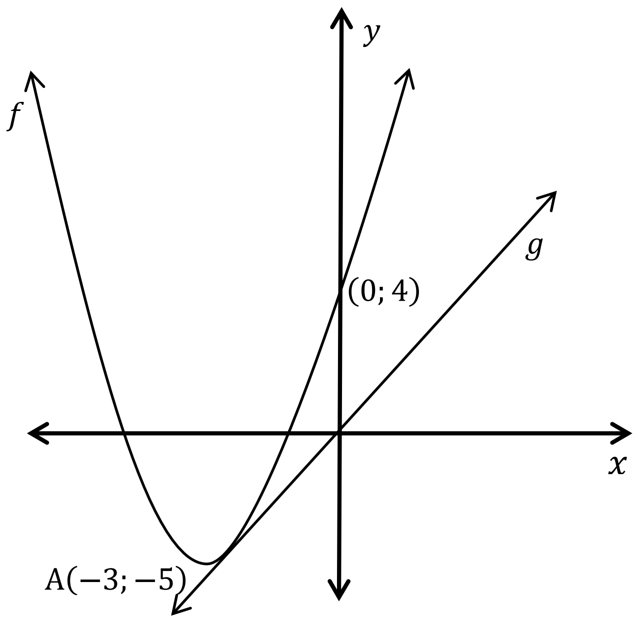
7.2.3 Write down the equation of the function g if g is the mirror image of f^{-1} in the y-axis. (2)

7.2.4 For which value(s) of x is: $f(x).f^{-1}(x) < 0$? (2)

[25]

QUESTION 8

The sketch below, which is not drawn to scale, shows the graph of $f(x) = ax^2 + bx + c$ and a straight line g, passing through the origin. The y-intercept of f is $(0; 4)$. Point A $(-3; -5)$ is the turning point of f.



8.1	Write down the equation of the axis of symmetry of f.	(1)
8.2	Show by calculation that $a = 1$ and $b = 6$.	(3)
8.3	Discuss the nature of the roots of f.	(3)
8.4	g is a tangent to f and the gradient of line g is 2. Determine the coordinates of the point of contact.	(4)
		[11]

QUESTION 9

9.1 Given: $f(x) = 3x^2 - 1$

Determine $f'(x)$ from first principles. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 5x^2 + \sqrt{x}$ (3)

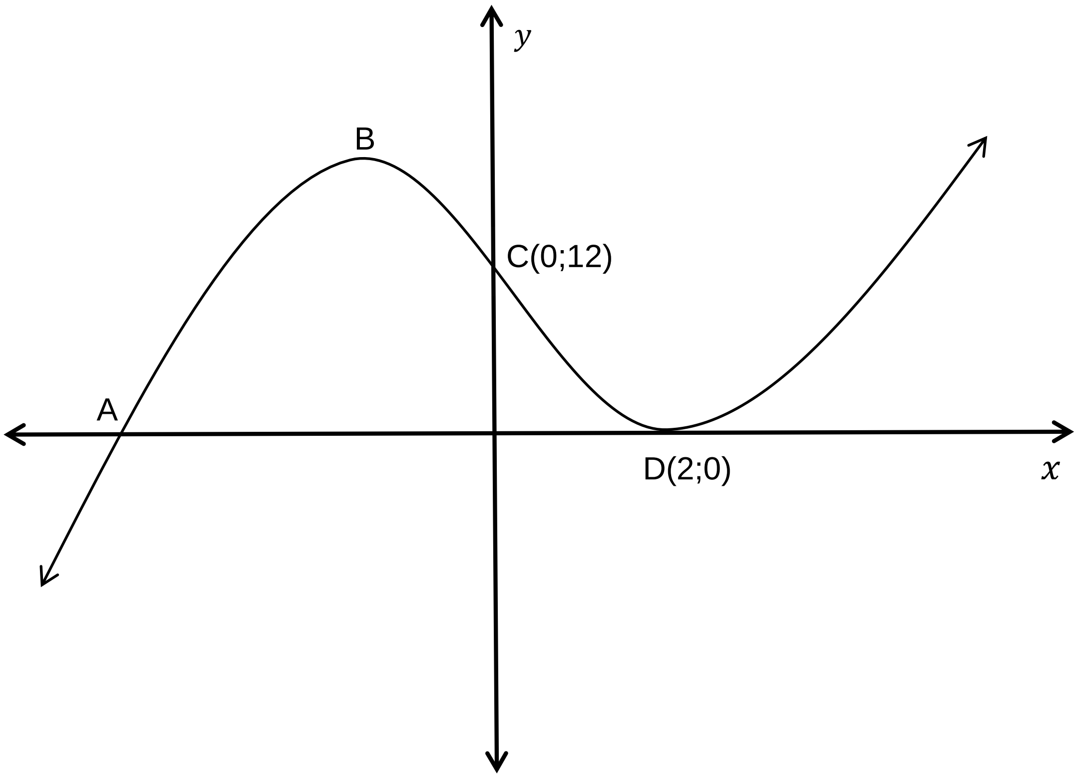
9.2.2 $D_x\left[\frac{6x-4}{3x}\right]$ (3)

9.3 Given $s(t) = t^3$. Show that the gradient of any tangent to s will never be negative. (2)

[13]

QUESTION 10

The sketch below shows the graph of the function $f(x) = x^3 - x^2 - 8x + 12$
A is an x-intercept and B a turning point of the graph. The points C$(0; 12)$ and D$(2; 0)$, the other turning point, are given on the sketch.



10.1 Determine the coordinates of A. (4)

10.2 Determine the coordinates of B. (5)

10.3 Calculate the x-coordinate of the point of inflection. (2)

10.4 Write down the value(s) of x for which $f'(x) > 0$. (2)

10.5 How many real roots will the equation $x^3 - x^2 - 8x + 12 = k$ have if $k < 0$? (2)

[15]

QUESTION 11

A water tank has both an inlet and an outlet which regulate the depth of water it contains. The depth D (in metres) is given by the function: $D(t) = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$ where D is measured in metres and t is measured in hours starting from 08h00.

11.1 Calculate the depth of the water at 08h00. (1)

11.2 Calculate the rate at which the depth is changing at 11h00. (3)

11.3 What happens to the depth of the water at 11h00? (1)

11.4 At what time will the inflow of water be the same as the outflow? (4)
[9]

QUESTION 12

12.1 A and B are mutually exclusive events. If it is given that the $\mathrm{P(A)} = 0{,}35$ and $\mathrm{P(B)} = 0{,}52$ determine:

12.1.1 $\mathrm{P(A')}$ (2)

12.1.2 $\mathrm{P(A\ and\ B)}$ (1)

12.1.3 $\mathrm{P(A\ or\ B)}$ (2)

12.2 Three married couples – Mr and Mrs Brown, Mr and Mrs Green and Mr and Mrs White are to be seated on a bench.

12.2.1 How many different arrangements are possible? (1)

12.2.2 If Mr and Mrs Green must be seated in the middle, how many different arrangements are possible for the remaining persons? (2)

12.2.3 Determine the probability that Mr and Mrs Green will sit next to each other. (4)
[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$ **;** $r \neq 1$ $S_\infty = \frac{a}{1 - r}$ **;** $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$